Decenber 3, 1997 WOCOMAL VARSITY MEET

ROUND I: Arithmetic – %, interest, discount, fractions and decimals

ALL ANSWERS MUST BE EXPRESSED AS DIRECTED IN THE PROBLEM

1. What is half of 2^{40} ? Express your answer as a power of 2.

2. If M is 30% of Q and Q is 20% of P and N is 50% of P, then what is the ratio of M to N in the simplest fractional form?

3. An industrial brush wears down linearly to $\frac{1}{4}$ of its original length in 30 days of regular use.

   After a new brush has been used daily for two weeks, what fractional part of the brush remains? Give your answer as a reduced fraction.

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, Bromfield, Shepherd Hill

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ROUND II: Algebra I – open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The product of $x + 2$ and $x - 2$ is how much less than the product of $x + 2$ and $x + 2$? Assume $x > 0$.

2. The sum of three numbers is 22. The ratio of the first to the third is 5:4 and the second is one more than the first. Find the largest of the three numbers.

3. Solve $\sqrt{2x+5} = 2\sqrt{2x} + 1$

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

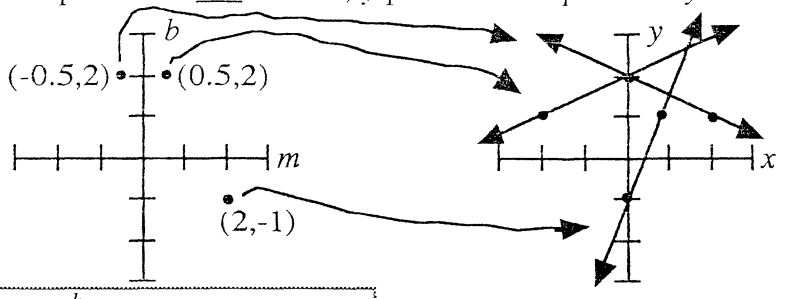
(3 pts) 3._____

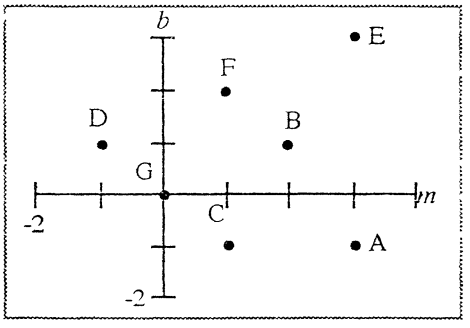
Algonquin, Shepherd Hill, South

ROUND III: Problem Solving

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

Consider two coordinate planes: a typical x, y plane and an m, b plane. Each point in the m, b plane corresponds to a line in the x, y plane with slope m and y–intercept b.

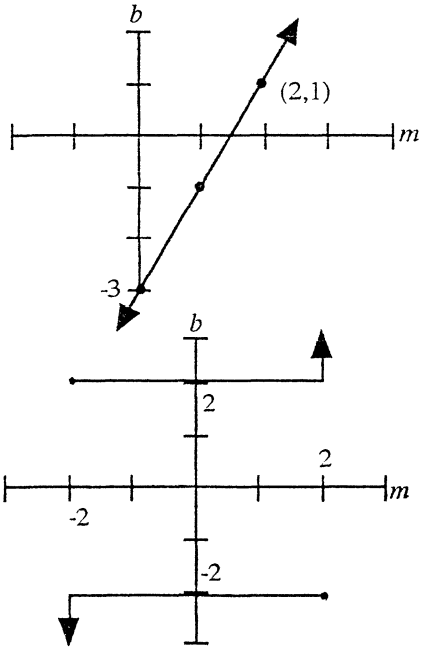


1. 

All the named point in the m, b plane to the left have integer coordinates. Which of them correspond to parallel lines?

2. The m,b line to the right corresponds to an infinite number of x, y lines. What point do they all have in common?

3. What is the area of the region left uncovered in the x, y plane when all the lines indicated to the right below are graphed?



ANSWERS

(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Mass Academy

ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Evaluate $\sum_{k=0}^{5}(k+1)^2$

2. The 10th and 13th terms of an arithmetic sequence are 5 and 8.75, respectively. How many terms of the sequence are negative?

3. A sequence $\{a_1, a_2, a_3, \ldots\}$ is defined by $a_1 = -5, a_2 = 4, a_n = a_{n-1} - a_{n-2}$ for $n > 2$.
   Find the sum $a_1 + a_2 + a_3 + \ldots + a_{1997}$

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Algonquin, Burncoat, Southbridge

ROUND V: Matrices

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The Wocomal Spa has two types of memberships, basic and extended. Matrix A gives the membership breakdown by type and gender at the year's start. Matrix B in the same format shows the new memberships during the year. Likewise, matrix C shows cancellations during the year.

$$A = \begin{matrix} \\ \text{basic} \\ \text{extended} \end{matrix}\begin{matrix} \begin{matrix} M & F \end{matrix} \\ \begin{bmatrix} 47 & 53 \\ 39 & 61 \end{bmatrix} \end{matrix} \qquad B = \begin{bmatrix} 15 & 13 \\ 14 & 9 \end{bmatrix} \qquad C = \begin{bmatrix} 21 & 18 \\ 19 & 11 \end{bmatrix}$$

Write a matrix showing spa memberships at the end of the year and label it in the way A is labeled.

2. Let T = [25 26 30 10 15] represent that a teacher has 5 classes with 25 students in the 1st, 26 in the 2nd, etc.
Also let $H = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ x & 0 \end{bmatrix}$ represent that she gave each student in the first class 3 handouts on Monday and 2 on Tuesday. Each student in the second class got 1 handout on Monday and none on Tuesday, etc. Find x if she gave out 126 more handouts on Monday than on Tuesday.

3. If $A = \begin{bmatrix} 3 & 1 \\ 4 & 7 \end{bmatrix}$, find a 2 by 2 matrix $B = \begin{bmatrix} & 1 \\ & \end{bmatrix}$ with three other positive integer entries, one of which is 2, so that $A \cdot B = B \cdot A$.

ANSWERS

(1 pt.) 1. $\begin{bmatrix} & \\ & \end{bmatrix}$

(2 pts) 2. $x =$

(3 pts) 3. $\begin{bmatrix} & \\ & \end{bmatrix}$

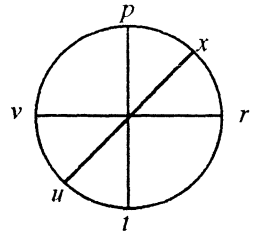
Decenber 3, 1997 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPERATE TEAM ROUND ANSWER SHEET 2 points each

1. The decimal $1.\overline{58}$ can be expressed as an improper fraction. When reduced to lowest terms the sum of the numerator and the denominator of this fraction is a perfect square. What is its positive square root?

2. What is the $y = mx + b$ form equation for the line containing the greatest number of points in the following list?
(–2,3), (1,2), (2,1), (3,0), (4,0), (5,1), (6,0), (8,-2)

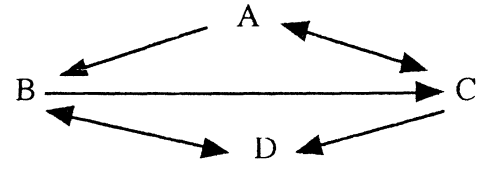
3. 

If letters at opposite end of a diameter represent reciprocals
($v = \frac{1}{r}$, for example)
and if $pr = x$, which of the following must be true? If none, say so, but there may be two or three.

A. $vt = u$ B. $pru = 1$ C. $p + r = v + t$

4. Beth has a new speed reading system for huge books. She starts at page 1, reads the whole page, then skips the amount of pages indicated by the number of the page she just read before reading another whole page. After "skipping" only ten times she arrives at the last page of the book, which is page number __?__.

5. Write a matrix that represents the communication network at the right.



6. Wanda noticed that her car's odometer displayed 081798. She wondered how many miles will have elapsed from the most recent previous palindrome display to the next occurence of a palindrome. Answer her question.

7. How many integer values of n are there so that $\frac{12}{n+2}$ is an integer?

8. Find the ordered pair of integers (x, y) which satisfies $36(5^x) = 225(4^y)$

9. The ordered list of numbers 18, 21, 24, a, 36, 37, b, has median 30 and mean 32. Evaluate $b - a$.

Algonquin, Assabet Valley, Burncoat, Hudson,
Mass Academy, St. John's, Shrewsbury, Westborough

December 3, 1007 — VOCONAL VARSITY MEET ANSWERS

ROUND I — arith

1. 1 pt 2^{39}
2. 2 pts $\frac{3}{25}$
3. 3 pts $\frac{13}{20}$

ROUND II — alg 1

1. 1 pt $4x+8$ OR $4(x+2)$
2. 2 pts 8.5 or $8\frac{1}{2}$
3. 3 pts $\frac{2}{9}$

ROUND III — prob solv

1. 1 pt A and E, C and F
2. 2 pts $(-2,-3)$
3. 3 pts 4

ROUND IV — seq series

1. 1 pt 91
2. 2 pts 5
3. 3 pts 9

ROUND V — matrix

1. 1 pt

$$\begin{matrix} & M & F \\ \text{basic} & 41 & 48 \\ \text{extended} & 34 & 59 \end{matrix}$$

need labels

2. 2 pts $x=5$
3. 3 pts $\begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}$

TEAM ROUND — 2 pts each

1. 16
2. $y=-\frac{1}{2}x+2$
3. A, B
4. 2,047
5.

from \ to	A	B	C	D
A	0	1	1	0
B	0	0	1	1
C	1	0	0	1
D	0	1	0	0

6. 1,100 miles
7. 12
8. (2, 1)
9. 28